EARTHQUAKE LOADING ON AXISYMMETRIC OFFSHORE STRUCTURES

by

Michael de St. Q. Isaacson
Department of Civil Engineering
University of British Columbia
Vancouver, B.C.

ABSTRACT

The calculation of base shear and overturning moment on a large (rigid) offshore structure is complicated by the inclusion of added mass and damping coefficients required to account for the structure's interaction with the surrounding ocean. Vertical axisymmetric structures have found a variety of applications such as oil storage tanks, production platforms and so on. The paper describes an efficient calculation procedure for determining the added mass and damping coefficients and consequently the earthquake loading of such structures. The approach used is based on a boundary element method involving an axisymmetric Green's function, and exploits the structure's axisymmetry to provide a highly efficient computational procedure suitable for carrying out on a desk-top computer. Results of maximum base shear and overturning moment are presented for a conical structure.

INTRODUCTION

The use of linear diffraction computer programs to calculate wave forces on large rigid offshore structures is now an established procedure in offshore design (eg. Sarpkaya and Isaacson, 1). The earthquake loading problem for such structures is directly related to the wave loading problem for structures undergoing motions, in that the same added mass and damping coefficients are required in order to account for the structure's interaction with the surrounding ocean.

Computer programs applicable to structures of arbitrary shape are quite costly. Alternative methods which are much more economical have been developed for more restricted configurations, in particular, vertical axisymmetric structures which have found various applications such as oil storage tanks and oil production platforms. This case has been treated by several authors in the context of wave loading, including

Fenton (2) and Isaacson (3), and the purpose of the present paper is to reconsider this case in the context of the earthquake loading problem. It is emphasized that a prescribed base motion is considered and the structure-foundation interaction problem is specifically not treated.

The approach used is based on a boundary element method involving an axisymmetric Green's function, and exploits the structure's axisymmetry to provide a highly efficient computational procedure suitable for carrying out on a microcomputer. The computational effort required is much less than for arbitrarily shaped bodies because of an extension to the analysis in which Fourier expansions are used to reduce the governing surface integral equation to a series of line integral equations, of which only one need be solved.

Results for a conical structure are presented, giving the relevant added mass and damping coefficient of the structure and the transfer functions relating base shear and overturning moment spectra to the base acceleration spectrum.

THEORETICAL DEVELOPMENT

A rigid vertical axisymmetric structure is resting on a horizontal seabed in water of constant depth, as depicted in Fig. 1(a), and is subjected to a unidirectional and sinusoidally varying base motion in the horizontal plane, given as $\xi \exp(-i\omega t)$, where t is time, ω is the angular frequency of the motion, and ξ is the complex amplitude of the motion. For simplicity, only a uni-directional motion is treated here, but the extension to several modes of base motion is straightforward by a consideration of the corresponding wave loading problem for a floating body (Ref. 3).

Let (r,θ,z) form a cylindrical coordinate system with z measured vertically upwards from the seabed along the structure's axis, r measured radially from the z axis, and θ measured from the direction of motion (see Fig. 1(a)). The fluid is assumed incompressible and inviscid and the flow irrotational. The fluid motion can thus be described by a velocity potential which satisfies the Laplace equation within the fluid region. In addition, the amplitude of motion is assumed sufficiently small for the boundary conditions at the water surface to be linearized. Consequently, the velocity potential is subject to the usual boundary conditions, linearized where appropriate on the seabed, the structure surface, the free surface and the far field.

The velocity potential is harmonic and proportional to the motion amplitude, and may thus be written as $\phi\xi \exp(-i\omega t)$. In the boundary integral method, the unknown potential $\phi(\underline{x})$ at the general point $\underline{x} = (r,\theta,z)$ is represented as due to a source distribution over the structure's surface S_o, and is thus expressed as:

$$\phi(\underline{x}) = \frac{1}{4\pi} \int_{S_o} f(\underline{X})\, G(\underline{x},\underline{X})\, dS \qquad (1)$$

Here $f(\underline{X})$ is a source strength distribution function, $G(\underline{x},\underline{X})$ is a Green's function for the general point $\underline{x}$ due to a source of unit strength at $\underline{X}$, and the integration is carried out for all points $\underline{X}$ over S_o. G is itself chosen to satisfy the Laplace equation, the seabed and linearized free surface boundary conditions, and the radiation condition. This ensures that ϕ also satisfies these equations, and it remains for f to be chosen so as to ensure that the boundary condition on the structure surface is satisfied.

This boundary condition equates the fluid velocity normal to the surface to the velocity of the surface itself in that direction, and reduces to:

$$\frac{\partial\phi}{\partial n} = - i\omega \cos\alpha \cos\theta \tag{2}$$

in which n, α and θ are defined in Fig. 1(a). This boundary condition, together with the representation for ϕ given in Eq. 1, gives rise to a surface integral equation for f:

$$- \frac{1}{2} f(\underline{x}) + \frac{1}{4\pi} \int_{S_o} f(\underline{X}) \frac{\partial G}{\partial n} (\underline{x},\underline{X}) \, dS = -i\omega \cos\alpha \cos\theta \tag{3}$$

Here n is measured from the point $\underline{x}$, and the integration is carried out over the point $\underline{X}$. In Eq. 3, $\underline{x}$ lies on the structure surface and may be defined by the coordinates (s,θ), where s is indicated in Fig. 1, and $\underline{X}$ may be defined by corresponding coordinates (s',θ').

Because of the structure's axisymmetry, the functions ϕ, f and G for points on the structure surface may be expanded as Fourier series:

$$\phi(s,\theta) = \sum_{m=0}^{\infty} \phi_m(s) \cos m\theta \tag{4}$$

$$f(s,\theta) = \sum_{m=0}^{\infty} f_m(s) \cos m\theta \tag{5}$$

$$G(s,\theta,s',\theta') = \sum_{m=0}^{\infty} G_m(s,s') \cos m(\theta-\theta') \tag{6}$$

and only the terms corresponding to $m=1$ will be required here. Substituting Eqs. 5 and 6 into Eq. 3, algebraic manipulation yields a set of line integral equations, of which the equation corresponding to $m=1$ is:

$$-f_1(s) + \frac{1}{2} \int_{S_o} f_1(s') \, R(s') \frac{\partial G_1}{\partial n} (s,s') \, ds' = - 2i\omega \cos\alpha \tag{7}$$

Here s_o is the structure's entire contour described by s, and $R(s')$ is the structure's radius at s'.

In a numerical solution to Eq. 7, the contour s_o is discretized into N short segments with the function f_1 taken to be uniform over each segment, and Eq. 7 is applied at the centre of each segment. Thus Eq. 7 may be approximated by a matrix equation:

$$\sum_{k=1}^{N} A_{jk}\, f_k^{(1)} = -2i\omega \cos \alpha_j \qquad \text{for } j = 1,2,\ldots N \tag{8}$$

where $f_k^{(1)}$ denotes $f_1(s_k)$. Expressions for the matrix coefficients A_{jk} are given by Fenton (2) and by Isaacson (3). Once the source strengths $f_k^{(1)}$ are determined, the potential itself can be obtained by a discretized form of Eq. 1. The necessary Fourier coefficient ϕ_1 at the j-th segment centre can be approximated as:

$$\phi_1(s_j) = \frac{1}{2}\sum_{k=1}^{N} f_k^{(1)}\, C_{jk} \qquad \text{for } j = 1,2,\ldots N \tag{9}$$

Once more, Fenton (2) and Isaacson (3) provide expressions for the coefficients C_{jk}.

Now that the potential function ϕ_1 is known, the hydrodynamic loads on the structure may be evaluated. The hydrodynamic pressure p acting on the structure surface is given by the linearized Bernoulli equation, $p = i\omega\rho\, \phi \exp(-i\omega t)$, where ρ is the fluid density. Thus the horizontal force $F_1^{(f)} \exp(-i\omega t)$ and overturning moment $F_2^{(f)} \exp(-i\omega t)$ due to the fluid may be expressed as:

$$F_j^{(f)} = -i\omega\rho \int_{S_o} \phi\, n_j\, ds\,, \qquad \text{for } j = 1,2 \tag{10}$$

where $n_1 = \cos\alpha \cos\theta$
$n_2 = z \cos\alpha \cos\theta - r \sin\alpha \cos\theta$

Substituting the Fourier expansion of ϕ, Eq. 5, and integrating with respect to θ, we obtain

$$F_j^{(f)} = -\pi\, i\omega\rho \sum_{k=1}^{N} L_k r_k n_{jk}\, \phi_1(s_k) \qquad \text{for } j = 1,2 \tag{11}$$

where L_k is the length of the k-th segment, and

$$n_{1k} = \cos \alpha_k$$

$$n_{2k} = z_k \cos(\alpha_k) - r_k \sin(\alpha_k)$$

The fluid forces $F_j^{(f)}$ are conveniently expressed in terms of added masses a_j and damping coefficients b_j by taking:

$$F_j^{(f)} = \omega^2 a_j + i\omega b_j \qquad (12)$$

in which a_j and b_j are real. Consequently, a_j and b_j may be retrieved by separating the real and imaginary parts of $F_j(f)$. It is emphasized that a_j and b_j are frequency dependent variables.

The remaining loads acting on the structure are the base shear $F_1 \exp(-i\omega t)$ and the overturning moment $F_2 \exp(-i\omega t)$ imposed by the structure's foundation, as indicated in Fig. 1(b). Thus the equations of motion of the structure may be expressed as:

$$F_j = \left[-\omega^2(m_j + a_j) - i\omega b_j\right]\xi \quad \text{for } j = 1,2 \qquad (13)$$

where m_1 is the mass of the structure and $m_2 = m_1 \ell$ with ℓ defined in Fig. 1(b). Now that the added mass and damping coefficients are known, Eq. 13 can be used to determine the base shear and overturning moment acting on the structure.

The extension to a random base motion is straightforward, with Eq. 13 providing the transfer functions relating the base shear and overturning moment spectra $S_{F_j}(f)$ to the base acceleration spectrum $S_{\ddot{\xi}}(f)$:

$$S_{F_j}(f) = |H_j(f)|^2 \, S_{\ddot{\xi}}(f) \quad \text{for } j = 1,2 \qquad (14)$$

with

$$H_j(f) = (m_j + a_j) + i b_j/\omega$$

Similarly, the base shear and overturning moment may also be obtained from a time record of the base motion.

RESULTS

A computer program based on the method outlined here has been tested by Isaacson (3) for a number of reference configurations in the context of wave loading. As an example of its application, it has been used here to generate results for the conical structure indicated in Fig. 2. The computed added mass and damping coefficients for this structure are shown as functions of frequency in Fig. 3. The computations were carried out at 10 discrete frequencies, and were all obtained with $N = 15$. In this case, a 15×15 matrix equation is solved for each computation in contrast to the very much larger matrix equation required in a program valid for arbitrary structure shape.

The corresponding transfer functions are given in Fig. 4. These may be used to obtain the base shear and overturning moment spectra for any design base acceleration spectrum.

CONCLUSIONS

The effect of a horizontal base motion on a rigid axisymmetric offshore structure is considered. The added mass and damping coefficients, which are required to account for the ocean-structure interaction, are obtained by a boundary method involving an axisymmetric Green's function. This enables the computation to be carried out much more efficiently than for the case of an arbitrary structural configuration.

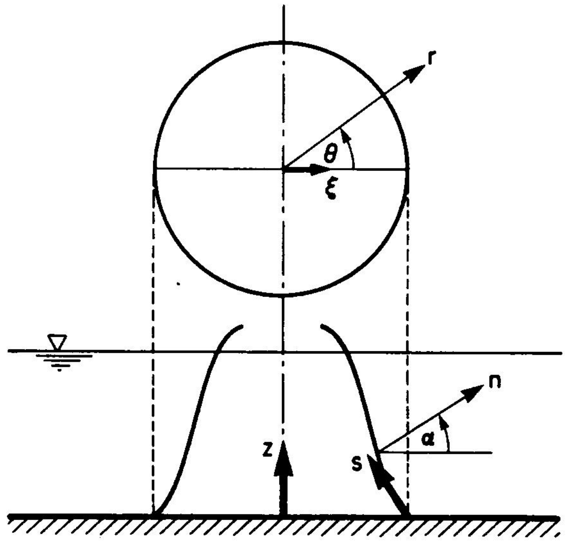
As an example of the method's application, results are presented for a conical offshore structure.

REFERENCES

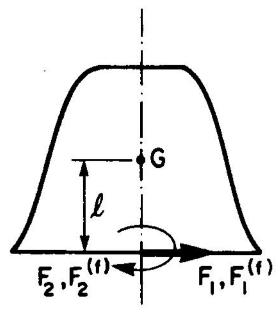
1. Sarpkaya, T., and Isaacson, M., Mechanics of Wave Forces on Offshore Structures, Van Nostrand Reinhold, New York, N.Y., 1981.

2. Fenton, J.D., "Wave Forces on Vertical Bodies of Revolution", Journal of Fluid Mechanics, Vol. 85, 1978, pp. 241-255.

3. Isaacson, M. de St. Q., "Fixed and Floating Axisymmetric Structures in Waves", Journal of the Waterway, Port, Coastal and Ocean Division, ASCE, Vol. 108, No. WW2, 1982, pp. 180-199.



(a) COORDINATE SYSTEMS



(b) EXTERNAL LOADS

Figure 1: Definition Sketch.

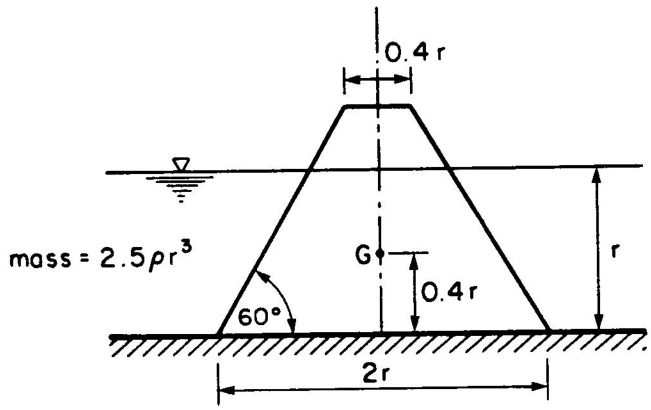
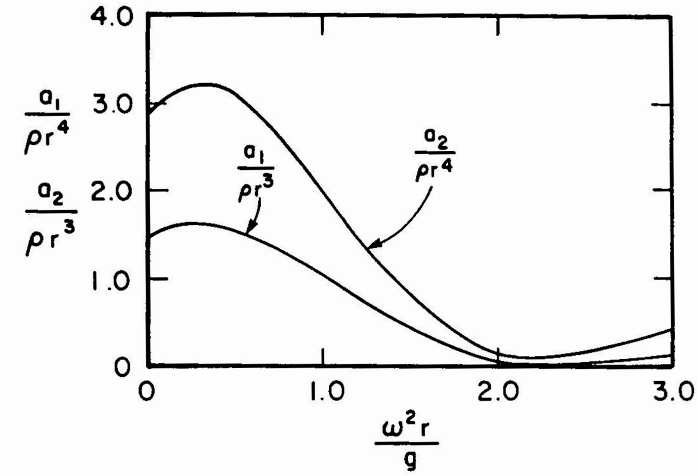
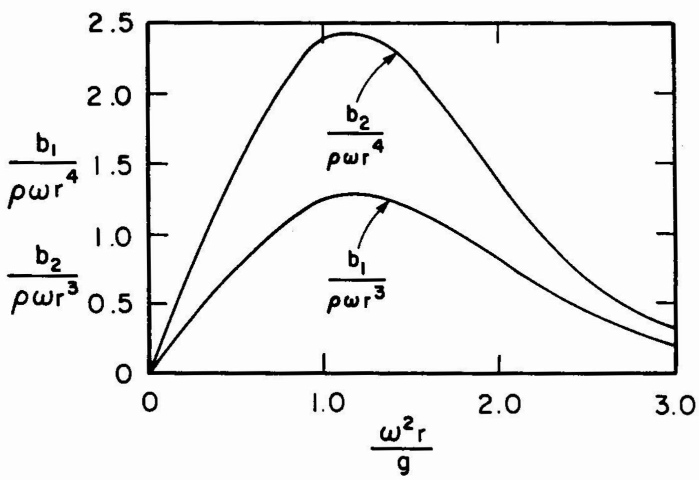


Figure 2: Conical Structure used as an Example.



(a) ADDED MASS COEFFICIENTS



(b) DAMPING COEFFICIENTS

Figure 3: Added mass and damping coefficients shown as functions of oscillation frequency. (r = base radius)

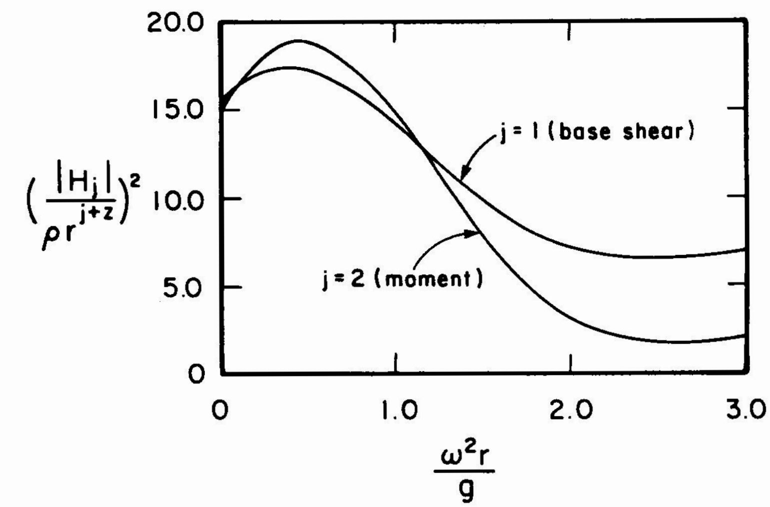


Figure 4: Transfer functions relating base shear and moment spectra to base acceleration spectrum.